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Balanced growth modelling of the link between COVID-19 cases, hospital admissions and deaths: Evidence from regions and age groups in England

Terence C. Mills

Loughborough Business School, Loughborough University, Ashby Road, Loughborough LE11 3TU, U.K.

Abstract

Analysing the mass of time series data accumulating from the COVID-19 coronavirus pandemic has become increasingly important as the pandemic has progressed through its numerous phases. This paper investigates the changing dynamic relationship between infections, hospital admissions and deaths using daily data from regions and age groups from England using balanced growth models. It is found that there has been a substantial decrease over time in the number of deaths and hospital admissions associated with an increase in infections as clinical practice has improved, and the vaccination program rolled out. These responses may be tracked and monitored through time to ascertain whether such improvements have been maintained.

Keywords: COVID-19; infections, admissions, and deaths; England; time series econometrics; balanced growth models.

1 Introduction

Since the first appearance of the COVID-19 coronavirus in early 2020 a vast research effort has been embarked upon examining the modelling and prediction of various aspects of what has become a global pandemic. Accessible reviews that concentrate on general features of this modelling are, for example, Vespignani et al (2020), Poletto, Scarpino and Volz (2020) and Gnanvi et al (2021), while discussion of the growth models widely used for predicting COVID-19 infections and deaths may be found in Tovissodé, Lokonen and Kakai (2020) and Shen (2020). Central to this modelling is the analysis of the mass of time series data accumulating both daily and weekly from the coronavirus pandemic and this has become ever more important as the pandemic has progressed through its numerous phases. Spiegelhalter and Masters (2021) provide an accessible introduction to such data issues, paying particular attention to the evidence emerging from the U.K.

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It is becoming increasingly apparent that econometric techniques are particularly suited to analysing this data: see, for example, Li and Linton (2020), Manski and Molinari (2020) and the review by Dolton (2021). Much of this research effort has focused on short-term forecasting of cases, hospital admissions and deaths, with notable examples being Doornik, Castle and Hendry (2020), Doornik, Hendry and Castle (2021) and Harvey, Kattuman and Thamotheram (2021). The research has also been directed at generalising, to stochastic settings, compartmental epidemiological models, such as the well-known ‘susceptible (S), infected (I) and recovered or deceased (R)’, or SIR, model, as in Korolev (2020) and Pesaran and Yang (2021).

The focus of the present paper is rather different, however, in that we investigate the changing dynamic relationship between infections, hospital admissions and deaths using daily data from England. Section 2 develops the recently proposed balanced growth model of Harvey (2020), Harvey and Kattuman (2020) and Mills (2022) for analysing the relationship between hospital admissions and subsequent deaths and the prior relationship between infections and subsequent hospital admissions, as well as proposing a method of linking the two models. Section 3 introduces the regional and age group data used in the analysis. The results for the regional data are reported and discussed in Section 4, while Section 5 reports a similar analysis and discussion for the age group data. A commentary on the findings is then provided in the concluding Section 6.

2 Balanced growth modelling of the relationship between infections, hospital admissions and deaths

2.1 Balanced growth modelling of the relationship between hospital admissions and subsequent deaths

Let daily deaths due to COVID-19 be denoted y_t, $t = 1, 2, \cdots, T$, with their cumulation being $Y_t = \sum_{j=1}^{t} y_j$, so that the growth rate of daily deaths is $g_{y,t} = y_t / Y_{t-1}$, $t = 2, 3, \cdots, T$. Similarly, denote daily hospital admissions due to COVID-19 by x_t, their cumulation by $X_t = \sum_{j=1}^{t} x_j$, and their growth rate by $g_{x,t} = x_t / X_{t-1}$,

Following Harvey and Kattuman (2020), we initially assume that there is balanced growth between daily deaths and hospital admissions lagged k days, which implies the regression model

$$\log(g_{y,t}) = \delta_y + \log(g_{x,t-k}) + \varepsilon_{y,t} \qquad t = k+1, \cdots, T \tag{1}$$

where $\varepsilon_{y,t}$ is an error term assumed to independently and identically distributed through time with zero mean and variance $\sigma^2_{y,\varepsilon}$, which is denoted $\varepsilon_{y,t} \sim \mathsf{IID}(0, \sigma^2_{y,\varepsilon})$. The ‘equilibrium’ relationship between the two growth rates is given by

$$g^*_{y,t} = \exp(\delta_y) g_{x,t-k}$$

Between daily deaths and admissions, y_t and x_t, the equilibrium is then

$$y^*_t = \exp(\delta_y)(Y_{t-1}/X_{t-k-1}) x_{t-k}$$

Allowing for a lag structure in the leading admissions series in (1) gives

$$\log(g_{y,t}) = \delta_y + \textstyle\sum_{i=h}^{k} \beta_i \log(g_{x,t-i}) + \varepsilon_{y,t} \tag{2}$$

where $h < k$ and $\sum_{i=h}^{k} \beta_i = 1$, a restriction that may be imposed by rewriting (2) as

$$\log(g_{y,t}) - \log(g_{x,t-k}) = \delta_y + \sum_{i=h}^{k-1} \beta_i \left(\log(g_{x,t-i}) - \log(g_{x,t-k})\right) + \varepsilon_{y,t}$$

so that $\beta_k = 1 - \sum_{i=h}^{k-1} \beta_i$, a restriction that ensures that there is indeed balanced growth. The corresponding equilibrium relationship is then

$$\bar{g}^*_{y,t} = \exp(\delta_y) \prod_{i=h}^{k} g_{x,t-i}^{\beta_i} = \exp(\delta_y) \bar{g}_{x,t-k}$$

where $\bar{g}_{x,t-k}$ is the weighted geometric mean of $g_{x,t-h}, \cdots, g_{x,t-k}$. The levels equilibrium is thus

$$\bar{y}^*_t = \exp(\delta_y)(Y_{t-1}/\bar{X}_{t-k-1}) \bar{x}_{t-k} = \Delta_y \bar{x}_{t-k}, \qquad t = k+1, k+2, \cdots, T$$

where $\bar{x}_{t-k}$ and $\bar{X}_{t-k-1}$ are the corresponding weighted geometric means of $x_{t-h}, \cdots, x_{t-k}$ and $X_{t-1-h}, \cdots, X_{t-1-k}$, respectively. Thus Δ_y measures the *long-run* response of deaths to an increase in hospital admissions: if daily admissions increase by 100 then deaths will increase by $100\Delta_y$ after k days.

When the two series are not on the same growth path, the model can be extended by replacing the intercept δ_y with a stochastic trend:

$$\log(g_{y,t}) = \delta_{y,t} + \textstyle\sum_{i=h}^{k} \beta_i \log(g_{x,t-i}) + \varepsilon_{y,t} \tag{3}$$

or

$$\log(g_{y,t}) - \log(g_{x,t-k}) = \delta_{y,t} + \textstyle\sum_{i=h}^{k-1} \beta_i \left(\log(g_{x,t-i}) - \log(g_{x,t-k})\right) + \varepsilon_{y,t} \tag{4}$$

where $\delta_{y,t}$ is defined as

$$\delta_{y,t} = \delta_{y,t-1} - \gamma_{t-1} + \eta_t \qquad \eta_t \sim \mathrm{IID}(0, \sigma^2_\eta)$$

$$\gamma_t = \gamma_{t-1} + \zeta_t \qquad \zeta_t \sim \text{IID}\left(0, \sigma_\zeta^2\right)$$

i.e., $\delta_{y,t}$ is a random walk with a drift that is itself potentially a random walk. If $\sigma_\zeta^2 = 0$ then $\gamma_t = \gamma_{t-1}$ and the drift is constant. On the other hand, if $\sigma_\eta^2 = 0$ then $\delta_{y,t} = 2\delta_{y,t-1} - \delta_{y,t-2} - \zeta_{t-1}$ and δ_t will tend to evolve very smoothly, being known as an integrated random walk (IRW). The equilibrium relationship in (4) is

$$\bar{g}_{y,t}^* = \exp\left(\delta_{y,t}\right)\bar{g}_{x,t-k}$$

so that the dynamic relationship between the two growth rates is given by $\exp\left(\delta_{y,t}\right)$. In terms of daily deaths and admissions, y_t and x_t, we have

$$\bar{y}_t^* = \exp\left(\delta_{y,t}\right)(Y_{t-1}/\bar{X}_{t-k-1})\bar{x}_{t-k} = \Delta_{y,t}\bar{x}_{t-k}$$

Thus, an increase of 100 in hospital admissions will lead to an increase of $100\Delta_{y,t}$ deaths in the following k days and this long run response will shift through time. As Harvey (2020) shows, this model may be arrived at by assuming that deaths and admissions follow Gompertz processes separated by k days, but such an assumption is not necessary.

A convenient way of fitting equation (4) is to cast it into state space form and employ the Kalman filter, which is part of the *Econometric Views* software package (all modelling presented in the paper was carried out using this package). The Kalman filter is a recursive procedure for computing the state vector, which contains the parameters $\delta_{y,t}$, and γ_t. Maximum likelihood estimates of the 'hyperparameters' σ_η^2 and σ_ζ^2 parameters may first be obtained using the predictive error decomposition form of the likelihood function constructed from the prediction errors from (4). The estimates of the δ parameters used to compute the Δ_t series shown in Figures 9 and 10 below are the 'smoothed' estimates, obtained by running the Kalman filter first forwards from $t = k + 1$ to $t = T$ and then backwards from $t = T$ to $t = k + 1$. Mills (2019, chapter 17) provides an introductory discussion to state space modelling and Harvey (1989) is the classic exposition.

2.2 Balanced growth modelling of the prior relationship between infections and hospital admissions and combining the two models

An analogous model may be developed for the prior relationship between infections (positive cases) and hospital admissions on denoting the growth rate of infections as $g_{w,t} = w_t/W_{t-1}$, where w_t and W_t are daily and cumulative infections, respectively. The balanced growth model is then

$$\log(g_{x,t}) - \log(g_{w,t-l}) = \delta_{x,t} + \sum_{i=m}^{l-1} \beta_i \left(\log(g_{w,t-i}) - \log(g_{w,t-l})\right) + \varepsilon_{w,t} \qquad (5)$$

where $\varepsilon_{w,t} \sim \mathrm{IID}(0, \sigma_{w,\varepsilon}^2)$ is an error term. The equilibrium response of admissions to infections is then

$$\Delta_{x,t} = \exp(\delta_{x,t})(X_{t-1}/\overline{W}_{t-l-1})$$

Given the 'causal structure' inherent in the relationship between infections, hospital admissions and deaths, the models (4) and (5) may be linked together. The response of deaths to an increase in infections is given by the product $\Delta_{y,t} \times \Delta_{x,t-k} = \Delta_t$ to ensure an appropriate timing match-up,

3 Regional and age group data and their manipulation

3.1 Regional time series

Daily data on case numbers (infections), hospital admissions and deaths have been made available by NHS England for the seven English regions: East, London, Midlands, North East, North West, South East, and South West. Figures 1-3 show this data from 20th March 2020 to 31st January 2022 (the data are taken from the UK government's COVID dashboard: soon after the end date this dashboard ceased publishing daily data on these variables). They follow a general pattern familiar to all who have been following the pandemic in England. Case numbers (see Figure 1) were relatively small during the first wave of the pandemic in the spring of 2020, a consequence of the limited testing that was then being carried out. With the widespread introduction of mass testing, case numbers in the second (October - November 2020) and third (December 2020 – January 2021) waves were considerably higher before declining during the spring and early summer of 2021. They then increased quickly during July before falling back and then stabilising at levels that were to remain through to November 2021. The advent of the 'omicron' variant then produced a major fourth wave in case numbers which was just beginning to tail off at the end of the sample period.

Nevertheless, there were regional differences, and not just in the number of cases, which partially reflect regional population sizes. The East, London and South East regions barely registered a second wave, which was most pronounced in the Midlands, North East and North West, confirming the 'north-south divide' of the pandemic during this period. The South West appears to be a consistent outlier, with a moderate second wave and large fluctuations in case numbers during the fourth wave, perhaps a consequence of localised difficulties in testing operations.

Hospital admissions (Figure 2) increased sharply in all regions during March 2020 before falling back to very low levels during the summer months. Following the increase in infections in the late summer, admissions began to rise during September, with the North East and North West showing a marked peak in late October – early November, but with only limited evidence of a second wave in the other regions. After a brief respite, admissions rose again, peaking at levels similar to those seen in the first wave during January 2021. After the spring and summer decline, admissions began to rise again during July before stabilising at levels much lower than those seen during the earlier waves. The omicron variant produced another wave in admissions in late December 2021 and January 2022, but the extent of the wave differed markedly across regions, with the North East and North West being particularly affected but the East, South East and South West experiencing only small increases in admissions. Deaths (Figure 3) follow a similar, although lagged, pattern to admissions but the numbers during the omicron wave have been at a much lower level than during previous waves, although the North East appears to be somewhat 'running hot'.

The logarithms of the daily growth rates of cases, admissions and deaths for each of the regions are shown in Figure 4, i.e., $\log(g_{w,t})$, $\log(g_{x,t})$ and $\log(g_{y,t})$. There are days on which there were zero deaths and, very occasionally, zero admissions, so that the logarithms of the associated daily growth rate are undefined. These missing values were 'filled in' by linear interpolation: their numbers are very small, less than four per cent of the total number of observations on deaths and admissions. While these log growth rates are clearly related in each region, they are equally clearly not on identical growth paths, so that models of the form (4) and (5) are necessary.

3.2 Age group time series

Daily age group data on hospital admissions are available from 13th October 2020 and this sets the start of the sample period for the age group analysis. The daily data is available in 5-year interval groups: 0-4, 5-9, 10-14, … , 85-89, 90+. For many of the younger age groups, however, daily deaths and hospital admissions, in particular, were either zero or very small and so the data was aggregated into the following seven age groups: 0-34, 35-44, 45-54, 55-64, 75-84 and 85+. Figure 5 shows the case numbers for each age group up to 31st January 2022, so that the first wave in the spring of 2020 and the summer hiatus of that year are excluded. All groups show the delta wave during the winter of 2020/2021 and the relatively minor wave during the autumn of 2021. The omicron wave during December 2021 and January 2022 is very pronounced. This pattern is reflected, in relative terms, for the hospital admissions data in Figure 6 and deaths in Figure 7. The logarithms of the daily growth rates of cases, admissions and deaths are shown in Figure 8. Missing values occur for deaths, mainly in the 0-34 and 35-44 age groups, but these only represent 11% of the total number of observations on deaths and were again filled in by linear interpolation.

4 Balanced growth modelling of the regional data

Before fitting the balanced growth models (4) and (5), the delay and lag parameters, h, j and k, l need to be chosen. Exploratory analysis and clinical considerations suggested that the settings $h = 0, k = 7$ in (4) and $j = 1, l = 14$ in (5) would be appropriate, i.e., it is possible that a patient admitted to hospital may die on the same day but, if they do die, will tend to die within seven days, and if an individual tests positive, the quickest they are admitted to hospital is after one day but, if they are admitted, it will tend to be within fourteen days.

Estimates of the two models using the sample period 20th March 2020 to 31st January 2022 for the regional data are reported in Table 1 in the form of standard deviations, i.e., σ_η, σ_ζ and σ_ε, and the 'signal to noise' error ratio $\sigma_\eta/\sigma_\varepsilon$, which provides a measure of the variability in the signal δ_t compared to the variability of the model error ε_t.

For the deaths equation (4), all regions are characterised by $\sigma_\zeta = 0$, so that the drift in the stochastic trend is constant, at $\gamma_t = \gamma$, say. Moreover, the estimates of γ were all found to be extremely small with large associated standard errors, so that the drifts are essentially zero and $\delta_{y,t}$ may be regarded as a driftless random walk for each region. Signal to noise error ratios lie in the range 0.2 to 0.4, with the North East and North West having relatively the smoothest stochastic trends.

In contrast, the admissions equation (5) has positive estimates of σ_ζ for all regions except the East and South West. For these two regions the constant drifts are insignificant, and their stochastic trends are thus driftless random walks. Figure 9 shows the estimates of $\Delta_{y,t}$, $\Delta_{x,t}$, and $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$ from 1st September 2020 to 31st January 2022, i.e., after the early part of the pandemic where there was very limited testing and after the summer 2020 hiatus.

The response of deaths to an increase in hospital admissions, $\Delta_{y,t}$ (the top panel of Figure 9), shows similar responses for all regions except London, which has a consistently low response until May 2021, and the South West, for which the response is unusually high in the early autumn of 2020 and the first half of 2021, this latter response possibly being a consequence of the later discovery that some 43,000 positive tests had been mis-classified as negative in this region during these months.

Regions: Estimates of equation (4)

Region	σ_η	σ_ζ	σ_ε	$\sigma_\eta/\sigma_\varepsilon$
East	0.0858 (0.0097)	0.0000 (4.7463)	0.3318 (0.0081)	0.26 (0.03)
London	0.1146 (0.0101)	0.0000 (12.286)	0.3163 (0.0077)	0.36 (0.04)
Midlands	0.0881 (0.0077)	0.0000 (4.1830)	0.2972 (0.0071)	0.30 (0.03)
North East	0.0656 (0.0072)	0.0000 (2.3330)	0.3107 (0.0064)	0.21 (0.02)
North West	0.0742 (0.0076)	0.0000 (2.6657)	0.3325 (0.0072)	0.22 (0.02)
South East	0.0913 (0.0099)	0.0000 (3.7377)	0.3228 (0.0078)	0.28 (0.03)
South West	0.1114 (0.0107)	0.0000 (5.9483)	0.3138 (0.0078)	0.35 (0.04)

Regions: Estimates of equation (5)

Region	σ_η	σ_ζ	σ_ε	$\sigma_\eta/\sigma_\varepsilon$
East	0.0893 (0.0078)	0.0000 (4.2836)	0.2564 (0.0060)	0.35 (0.03)
London	0.0759 (0.0062)	0.0036 (0.0006)	0.1969 (0.0042)	0.39 (0.04)
Midlands	0.0674 (0.0057)	0.0017 (0.0008)	0.1588 (0.0036)	0.42 (0.04)
North East	0.0618 (0.0056)	0.0024 (0.0009)	0.1722 (0.0037)	0.36 (0.04)
North West	0.0886 (0.0053)	0.0008 (0.0009)	0.1737 (0.0042)	0.51 (0.04)
South East	0.0646 (0.0077)	0.0019 (0.0006)	0.2427 (0.0052)	0.27 (0.03)
South West	0.1171 (0.0082)	0.0001 (0.0029)	0.2869 (0.0067)	0.41 (0.03)

Table 1 Estimates of equations (4) and (5) for the English regions (standard errors in parentheses): 21st March 2020 31st January 2022.

A noticeable feature of these estimates of $\Delta_{y,t}$ is that, since June 2021, the regional responses have both declined and converged, even during the omicron wave, lying in the range 0.10 (London) to 0.16 (North West) by the end of January 2022 (i.e., 10 to 16 additional deaths in response to an increase of 100 admissions), compared to a range of between 0.2 to over 0.4 at the beginning of 2021, so that here has been more than a 50% decline in the 'death rate'.

The regional responses of hospital admissions to an increase in cases (infections), the middle panel of Figure 9, also show both declines and convergence over time. In the middle of December 2020, $\Delta_{x,t}$ lay in the range 0.087 to 0.150 (i.e., 87 to 150 additional admissions as a result of a 1000 case increase); by the end of January 2022, this range was 0.012 to 0.020 (12 to 20), an approximately 85% decline in admissions. The response of deaths to an increase in cases, $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$, is shown in the bottom panel of Figure 9. Again, there are both declines and convergence in regional responses from the summer of 2021, all regions having responses between 0.002 and 0.003 by the end of January 2022 (between 2 and 3 additional deaths as a result of an additional 1000 cases), compared to around 0.03 (30) at the beginning of 2021, representing a 90% decline.

5 Balanced growth modelling of the age group data

Table 2 reports estimates of models (4) and (5) for the age group data from 13th October 2020 to 31st January 2022. For all age groups the estimates of σ_η and σ_ζ are positive, usually reliably so, in both equations. The estimates of $\Delta_{y,t}$, $\Delta_{x,t}$, and $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$ from 13th October 2020 to 31st January 2022 are shown in the top, middle and bottom rows of Figure 10. To aid clarity, each row contains two graphs, the left hand graph giving the estimates for the four groups that are aged less than 65, the right hand graph giving the estimates for the three groups aged 65 and over .

For all three responses, the size of the response increases with age. Under the age of 65, $\Delta_{y,t}$ remains relatively constant throughout the sample period, apart from marked increases during the early summer, which are primarily a consequence of the major decline in hospital admissions for these age groups during this period. In contrast, the over-75s show major declines in $\Delta_{y,t}$ during February to April 2021.

At the end of December 2020, $\Delta_{y,t}$ stood at 0.019 for the 0-34 age group (an additional 2 deaths for an increase in admissions of 100), 0.038 (4) for the 35-44 age group, 0.064 (6) for the 45-54 age group and 0.104 (10) for the 55-64 age group. By the end of January 2022 these values stood at 0.006 (1), 0.020 (2), 0.071 (7) and 0.103 (10). On 28th January 2021, $\Delta_{y,t}$ values for the over 65s peaked at 0.32, 0.46 and 0.64 for the 65-74, 75-84 and 85+ age groups. By the end of January 2022, these values had fallen to 0.16, 0.22 and 0.36, thus producing reductions of 16, 24 and 28 deaths per 100 admissions.

$\Delta_{x,t}$ has remained relatively stable for most of the sample period for the two youngest age groups: the mean value of $\Delta_{x,t}$ for the 0-34 age group is 0.011 (11 additional hospital admissions for an increase of 1000 cases) and for the 35-44 age group it is 0.024 (24 per thousand); nevertheless, both have seen declines over the last few months, for by the end of January 2022 the values for the two groups were 0.005 (5) and 0.006 (6). For the older groups there has been sustained falls in $\Delta_{x,t}$ since the winter of 2021: the 55-64 age group has seen a fall from around 0.11 (110 admissions per thousand cases) to 0.02 (20 per thousand) by the end of January 2022; for the 65-74 age group the fall has been from 0.35 (350 per 1000) to 0.07 (70 per thousand); the 75-84 age group has seen falls from 0.55 (550 in thousand) to 0.22 (220 per thousand), while for the over-85s the fall has been from 0.70 (700 per thousand) to 0.28 (280 per thousand).

Age groups: Estimates of equation (4)

Age group	σ_η	σ_ζ	σ_ε	$\sigma_\eta/\sigma_\varepsilon$
$0-34$	0.0995 (0.0161)	0.0014 (0.0010)	0.4093 (0.0144)	0.24 (0.04)
$35-44$	0.0892 (0.0149)	0.0016 (0.0010)	0.4326 (0.0160)	0.21 (0.04)
$45-54$	0.1198 (0.0149)	0.0016 (0.0014)	0.3910 (0.0112)	0.31 (0.04)
$55-64$	0.1094 (0.0142)	0.0018 (0.0011)	0.3292 (0.0105)	0.33 (0.05)
$65-74$	0.0665 (0.0113)	0.0032 (0.0011)	0.3076 (0.0072)	0.22 (0.04)
$75-84$	0.0355 (0.0168)	0.0038 (0.0014)	0.2853 (0.0066)	0.12 (0.06)
$85+$	0.0995 (0.0091)	0.0026 (0.0013)	0.2650 (0.0062)	0.34 (0.04)

Age groups: Estimates of equation (5)

Age group	σ_η	σ_ζ	σ_ε	$\sigma_\eta/\sigma_\varepsilon$
$0-34$	0.0634 (0.0062)	0.0019 (0.0011)	0.1250 (0.0046)	0.51 (0.06)
$35-44$	0.0412 (0.0101)	0.0047 (0.0012)	0.1639 (0.0055)	0.25 (0.07)
$45-54$	0.0519 (0.0084)	0.0041 (0.0013)	0.1505 (0.0050)	0.35 (0.06)
$55-64$	0.0535 (0.0091)	0.0041 (0.0015)	0.1444 (0.0039)	0.37 (0.07)
$65-74$	0.0575 (0.0086)	0.0051 (0.0018)	0.1434 (0.0051)	0.40 (0.07)
$75-84$	0.0331 (0.0153)	0.0065 (0.0017)	0.1535 (0.0044)	0.22 (0.10)
$85+$	0.0614 (0.0099)	0.0051 (0.0017)	0.1491 (0.0053)	0.41 (0.08)

Table 2 Estimates of equations (4) and (5) for English age groups (standard errors in parentheses): 13th October 2020 – 31st January 2022.

As a consequence of these movements, $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$ has remained stable at 1 death per 10,000 cases for the under-35s and 4 deaths per 10,000 cases for the 35-44 age group. For the 45-54 age group the death rate has dropped from a peak of 12 per thousand to 1 per thousand by the end of January 2022; for the 55-64 age group the drop has been from a peak of 17 per thousand to 2 per thousand; for the 65-74 age group it has declined from a peak of 93 per thousand to 10 per thousand; for the 75-84 age group the decline has been from a peak of 292 per thousand to 45 per thousand; and for the over-85s the decline has been from a peak of 425 per thousand to 100 per thousand.

6 Commentary

The fitting of balanced growth models containing stochastic trends to regional and age-group data on infections, hospital admissions and deaths using daily data that includes the omicron wave has produced some interesting and important findings. Both the regional and age group data have shown falls in deaths and admissions for a given increase in cases over time and this is likely to have been a consequence of the roll-out of the vaccination and booster programme that commenced in early 2021. These falls started earlier and have been most pronounced in the older age groups, and least noticeable in the youngest groups, as the latter had yet to see a significant vaccine take up by the end of the sample period. Regional variations in admission and death rates have declined over time, with the marked lead of the London region in terms of relatively small death and admissions rates being reduced as, presumably, clinical practices in the rest of the country have caught up with the initially more efficient practices of the capital.

It is straightforward to update these models as more data arrive: indeed, this has been done regularly throughout the last few months of the pandemic. This can then provide real-time information on whether or not the improvements in clinical procedures and vaccine protection are being maintained, thus giving timely indicators of whether policies that allow for 'living with COVID' need to be updated.

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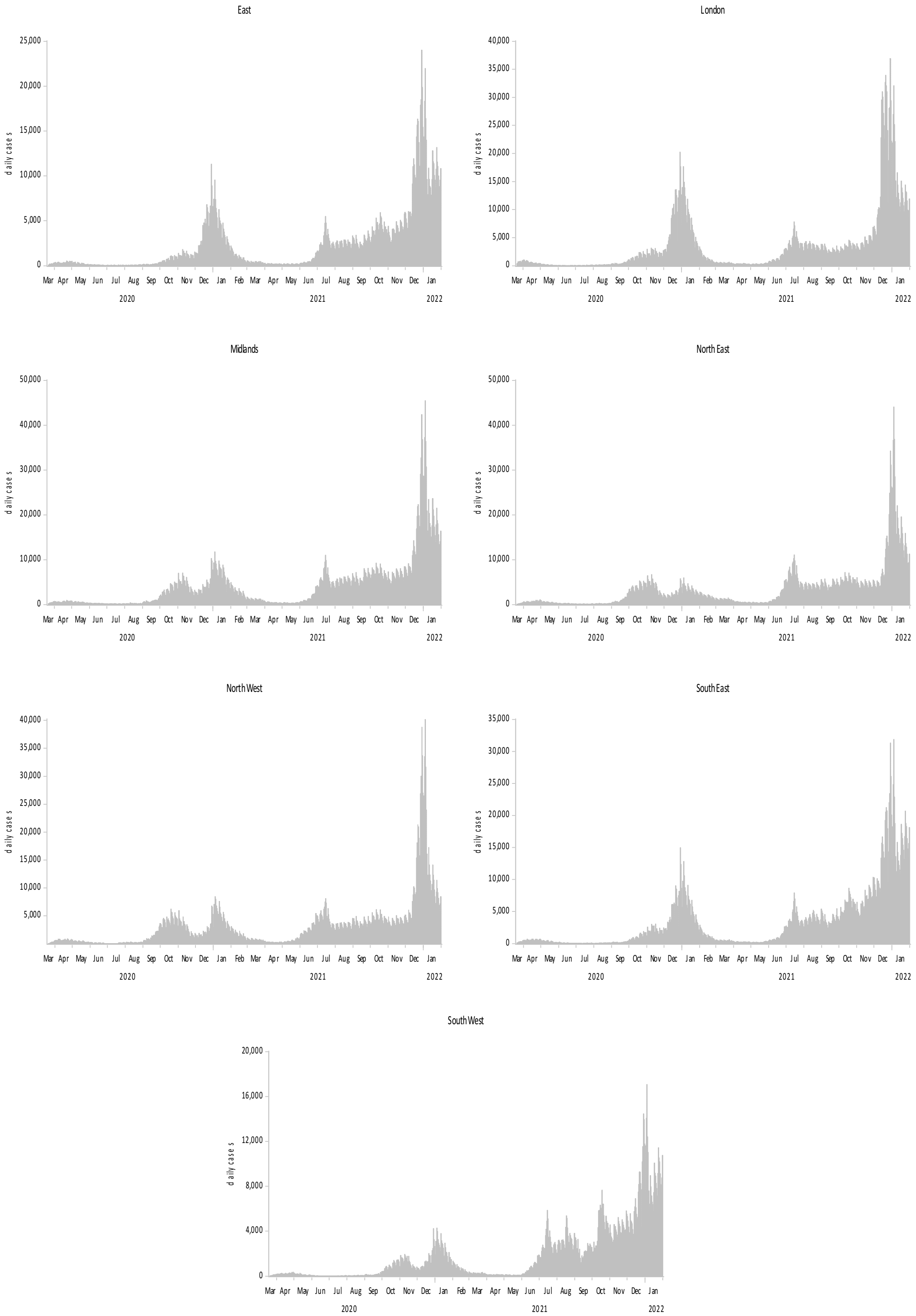

Figure 1 Daily cases for the English regions: 21st March 2020 – 31st January 2022.

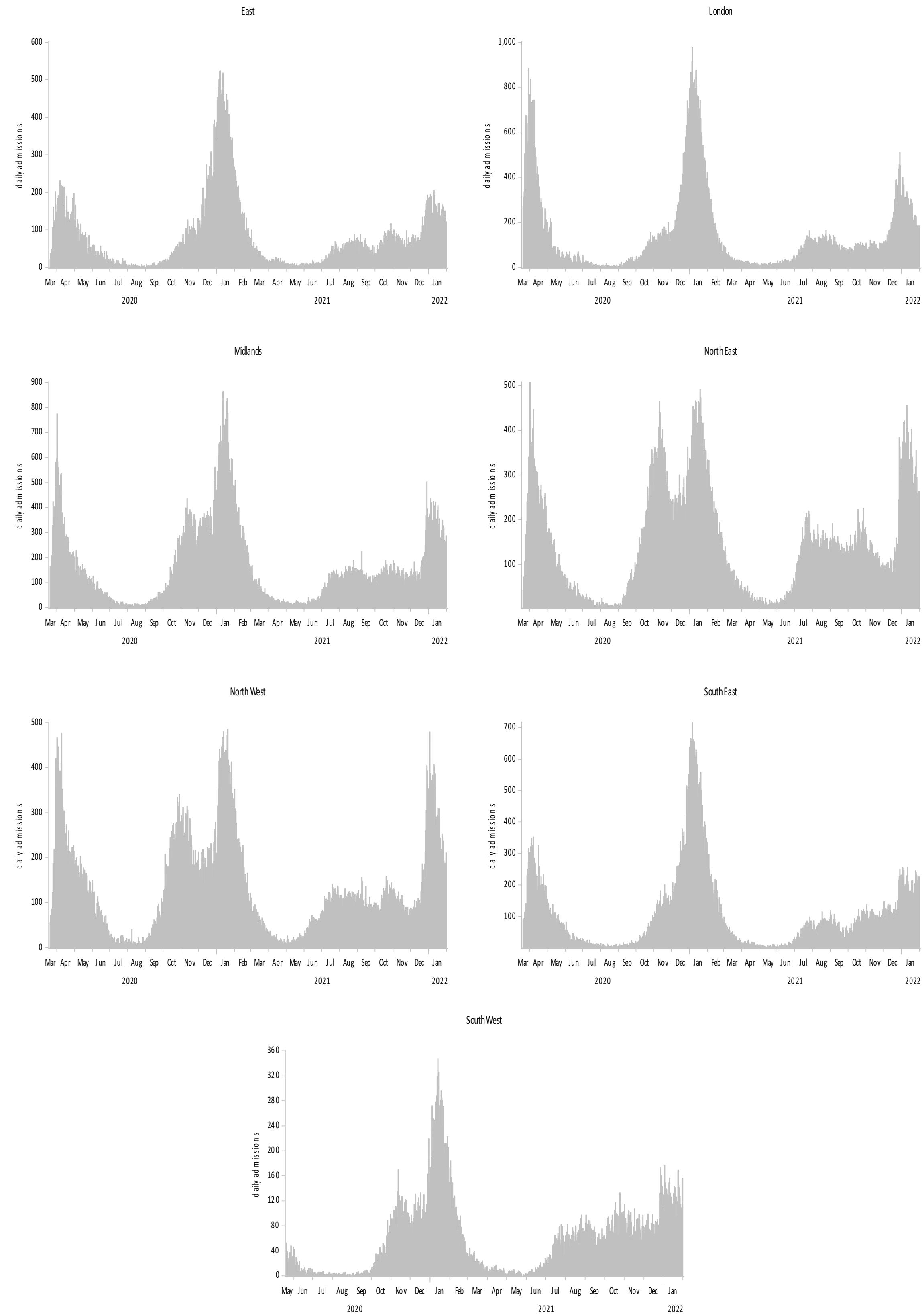

Figure 2 Daily admissions for the English regions: 21st March 2020 – 31st January 2022.

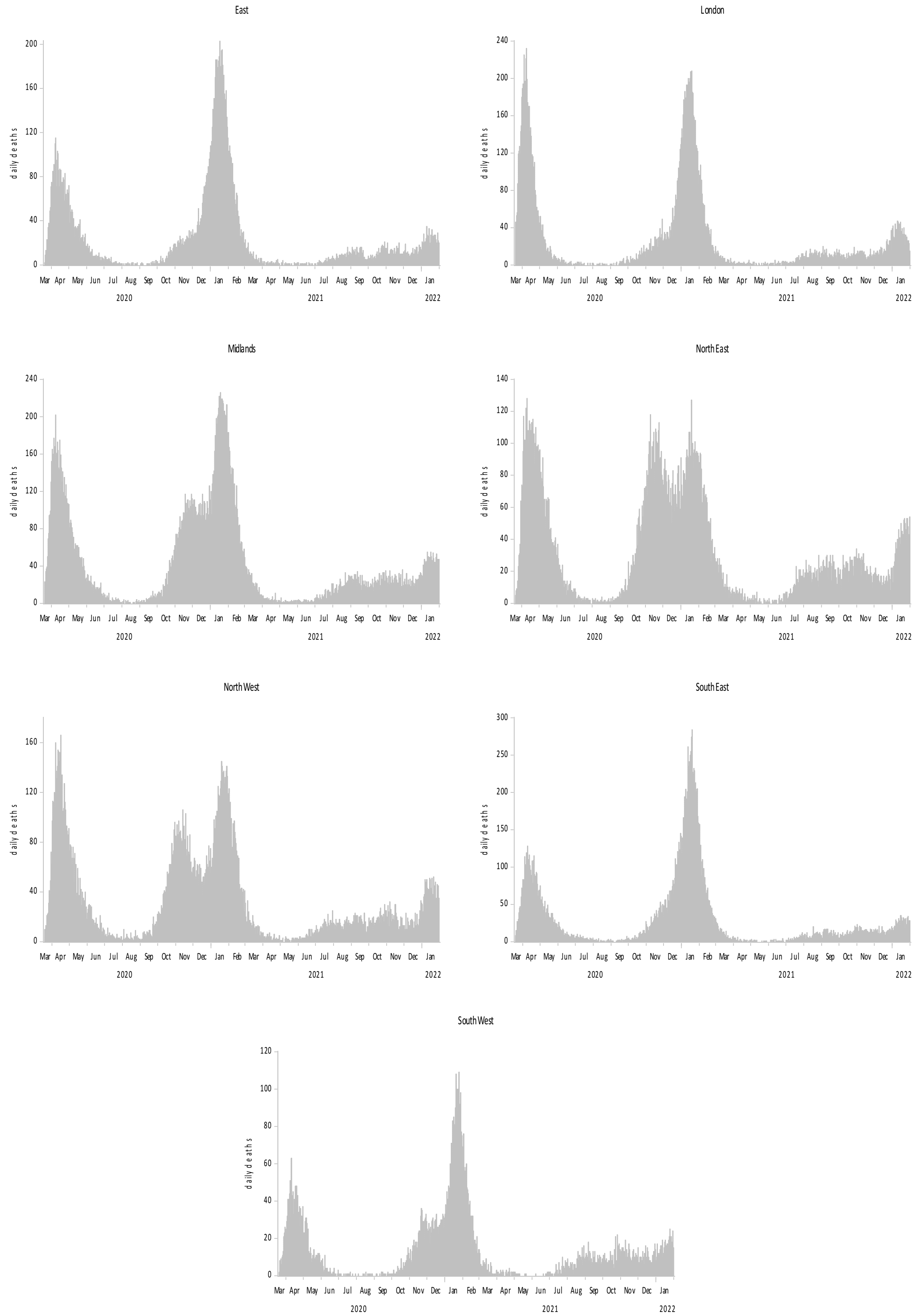

Figure 3 Daily deaths for the English regions: 21st March 2020 – 31st January 2022.

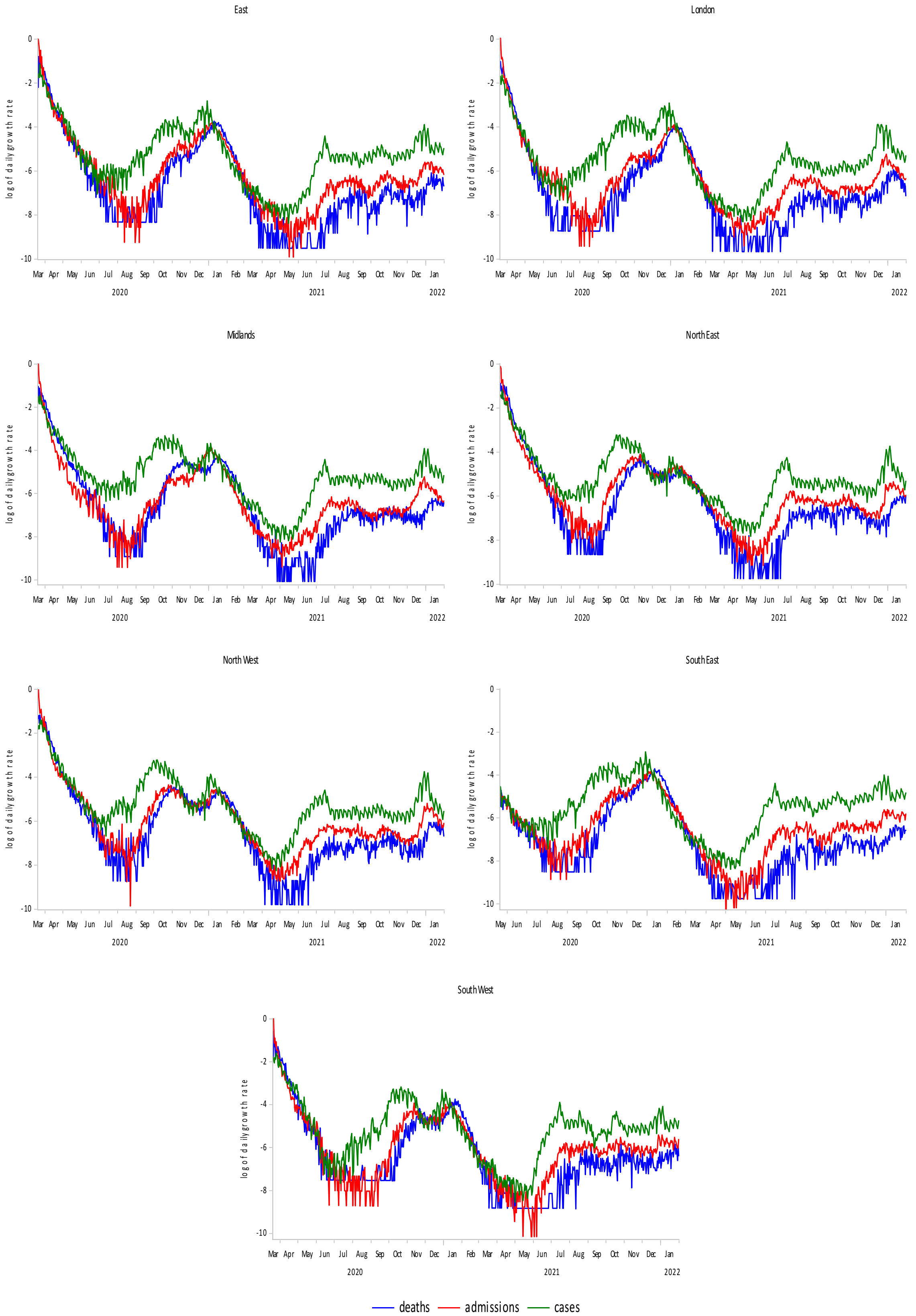

Figure 4 Logs of daily growth rates for English regions: 21st March 2020 – 31st January 2022

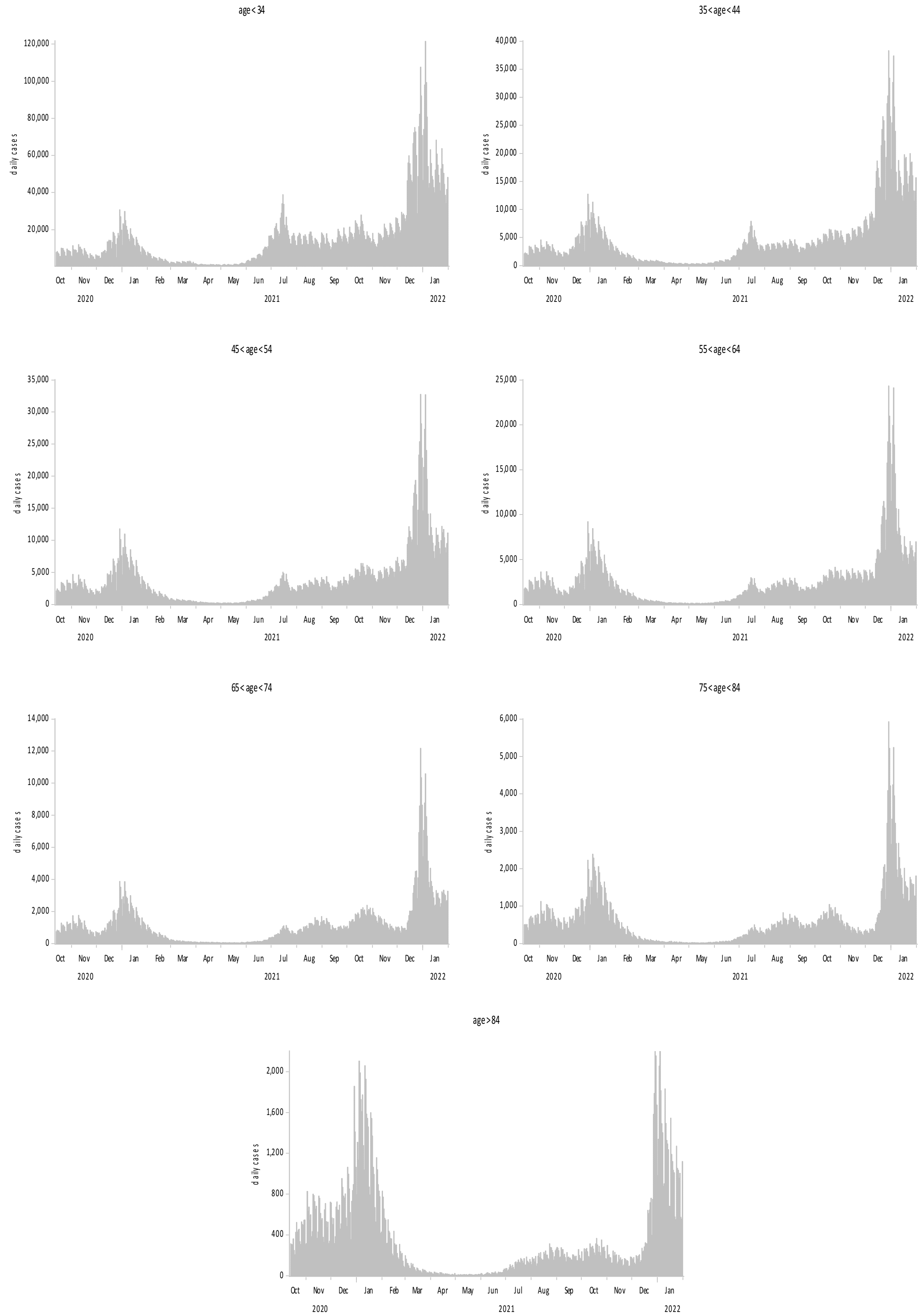

Figure 5 Daily cases for the English age groups: 13th October 2020 – 31st January 2022.

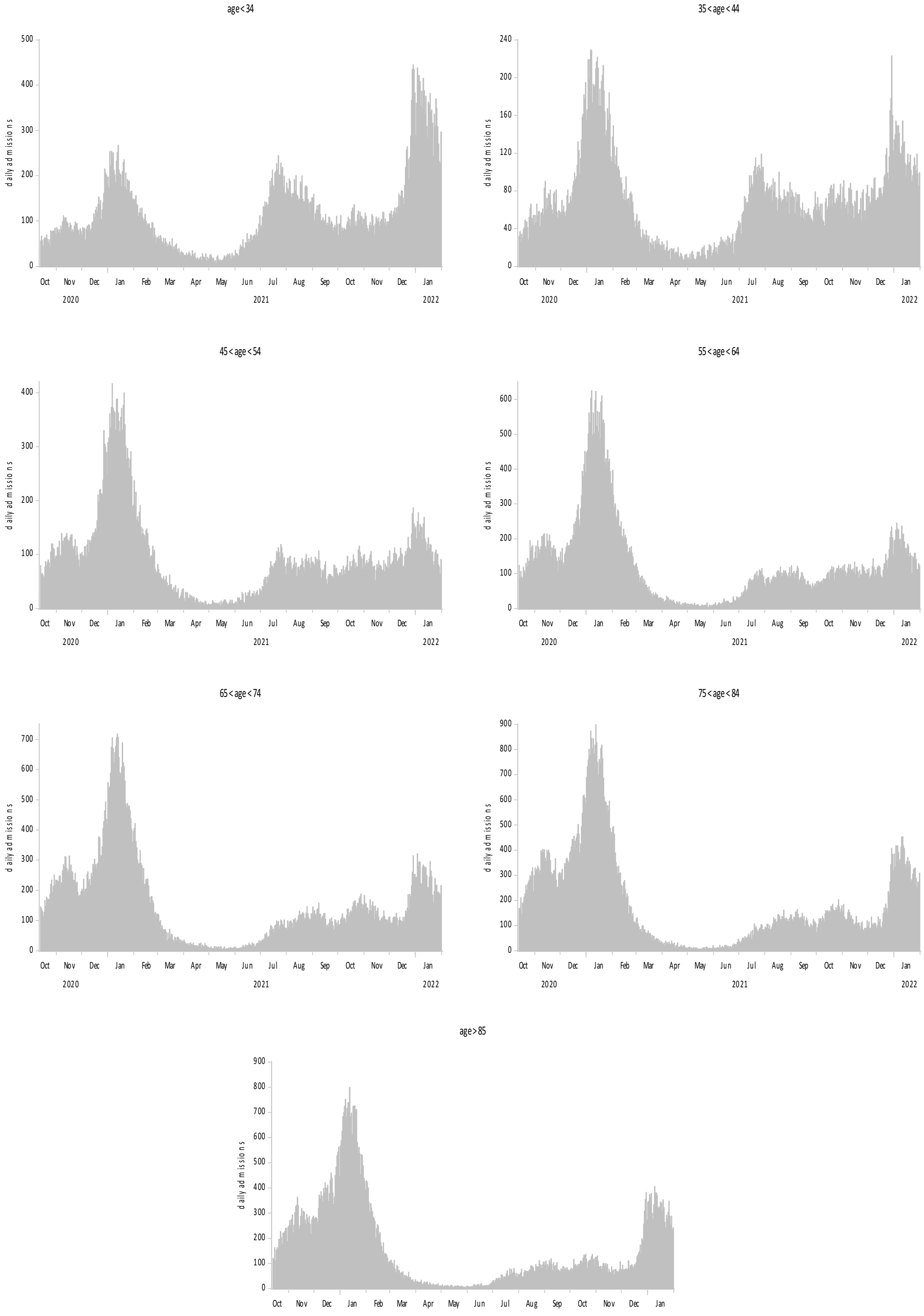

Figure 6 Daily admissions for the English age groups: 13th October 2020 – 31st January 2022.

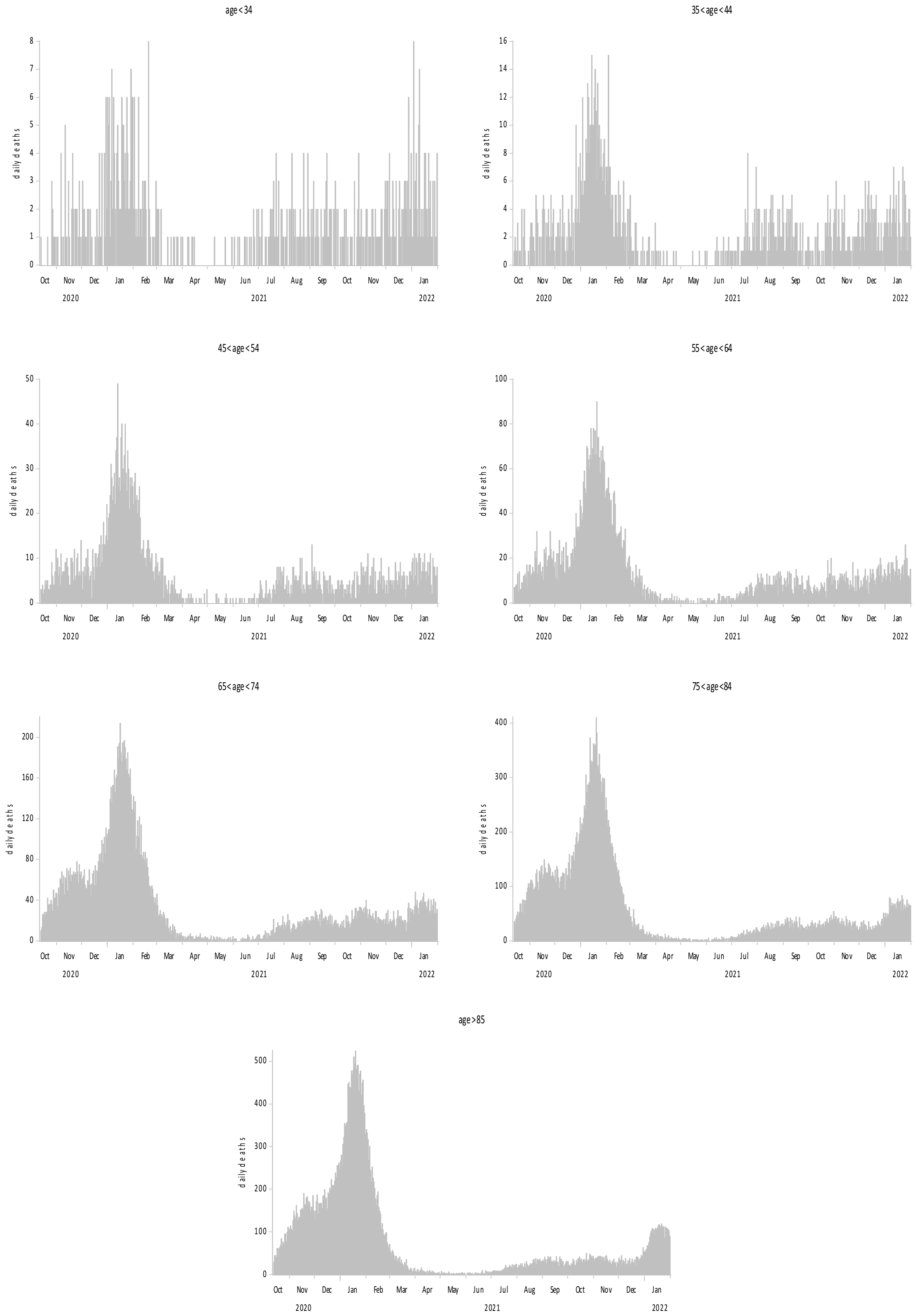

Figure 7 Daily deaths for the English age groups: 13th October 2020 – 31st January 2022.

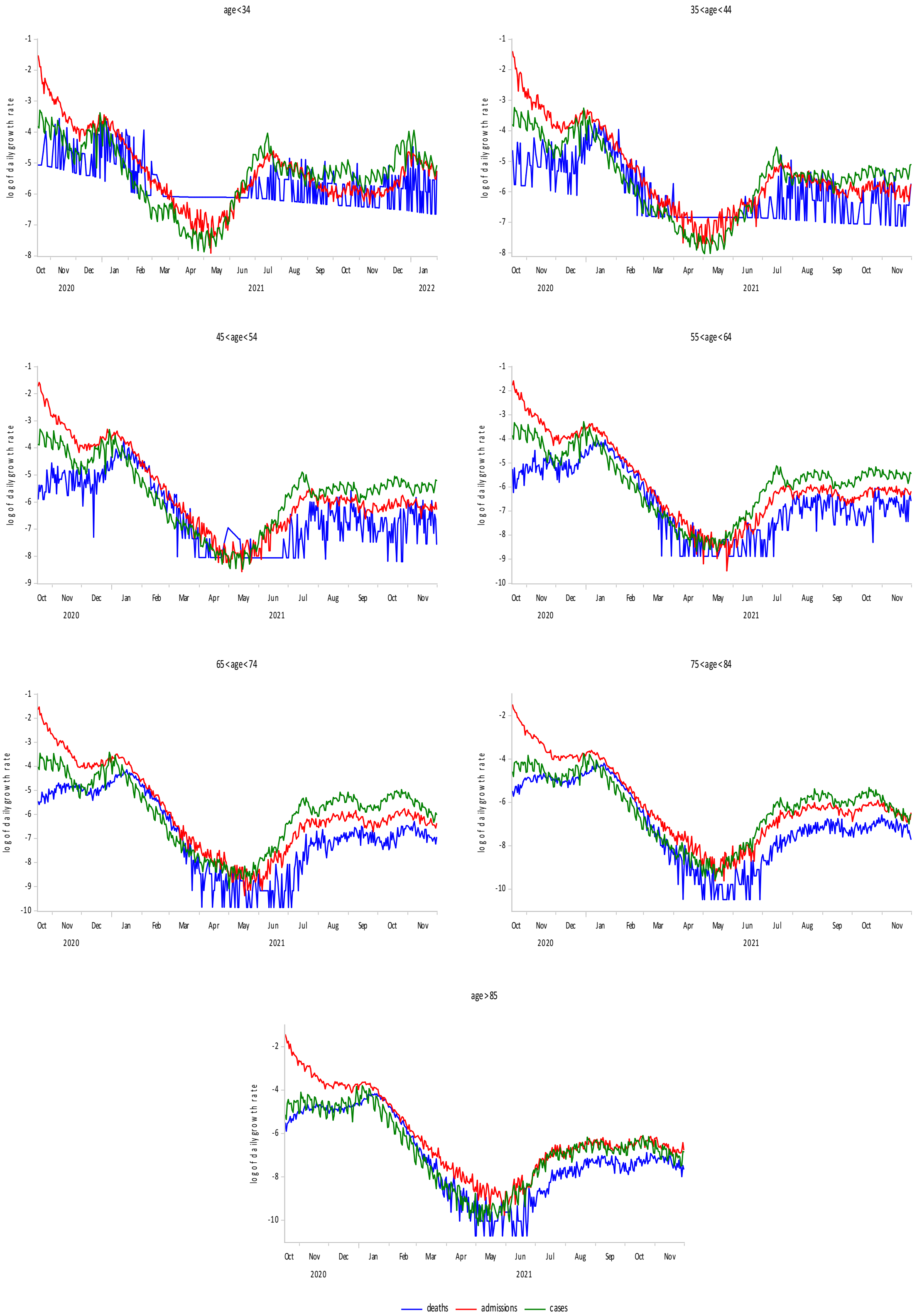

Figure 8 Logs of daily growth rates for English age groups 13th October 2020 – 31st January 2022.

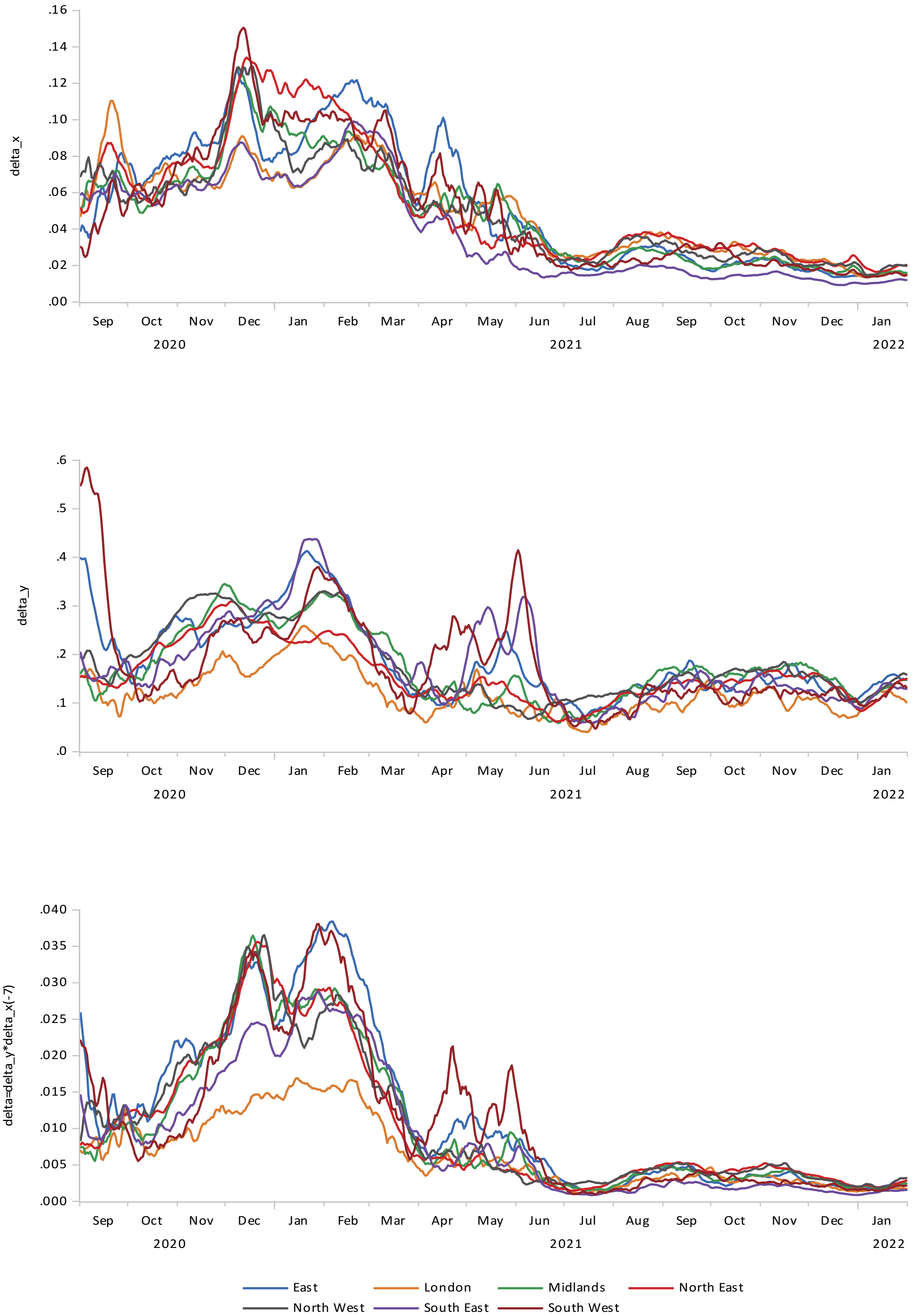

Figure 9 Estimates of $\Delta_{x,t}$ (top panel), $\Delta_{y,t}$ (middle panel), and $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$ (bottom panel) for English regions: 1st September 2020 – 31st January 2022.

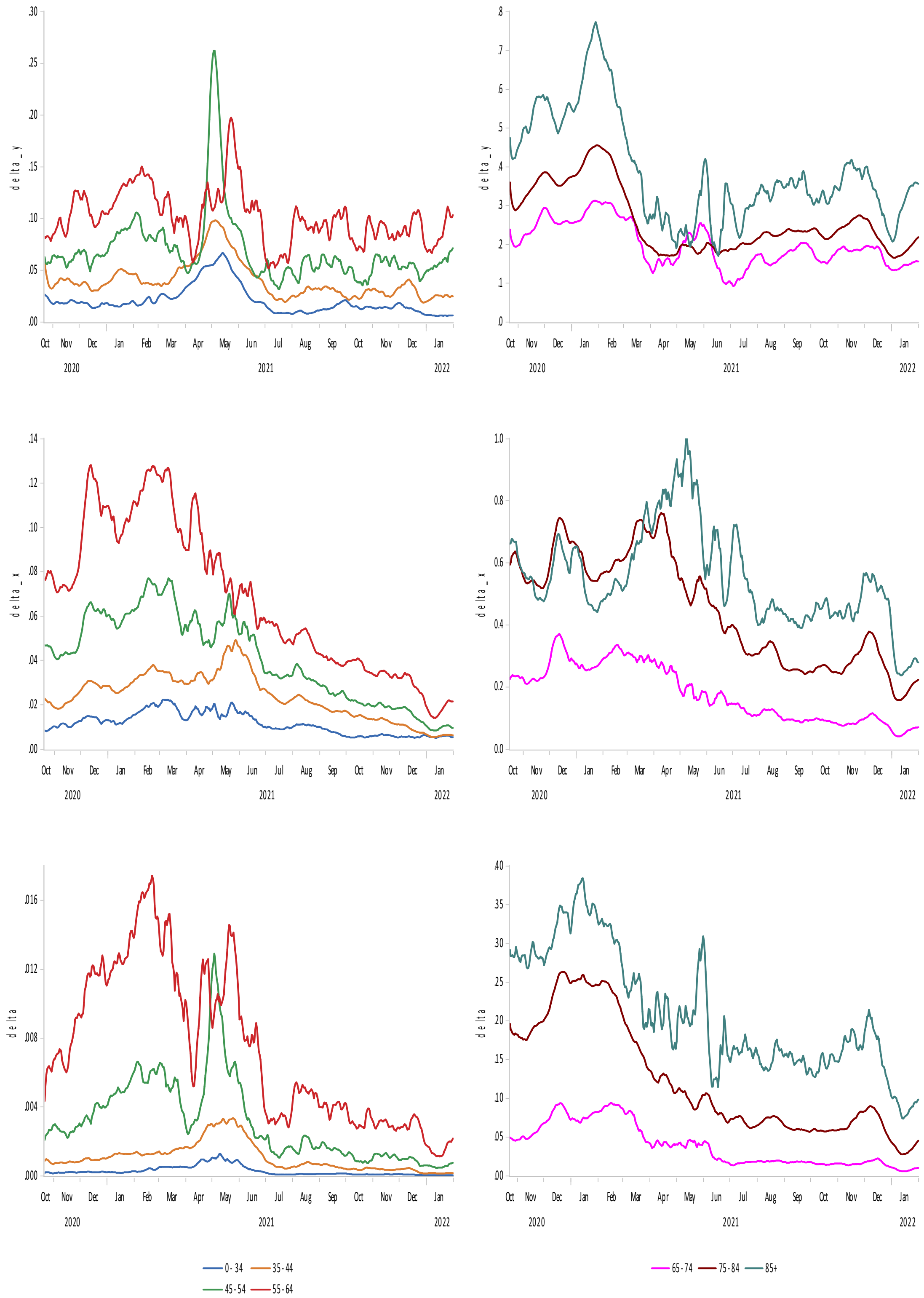

Figure 10 Estimates of $\Delta_{y,t}$ (top panel), $\Delta_{x,t}$ (middle panel), and $\Delta_t = \Delta_{y,t} \times \Delta_{x,t-7}$ (bottom panel) for English age groups: 13th October 2020 – 31st January 2022.